

Conference Paper

Design of Free Vibration Single Degree of Freedom Horizontal Bending Bar Testing Equipment

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ABSTRACT

Vibration is a subdiscipline of dynamics, which is the study of the repetitive motion of objects relative to stationary frames of reference. The process of learning vibration science in universities can take place through practicum activities. According to the learning experience, the single degree of freedom horizontal bending bar is constructed. The equipment can conduct free vibration in several variations to give students different approaches. The objective of the practicum is to find the value of natural frequency. This paper uses three methods to obtain the natural frequency: Rayleigh method, software, and experiments. The other variables that influence the natural frequency are the length of the bar and its material. By comparing the Rayleigh method with software, the natural frequency value with Copper, Brass, and Stainless-Steel material bars decreased by 3.379 percent, 8.735 percent, and 1.765 percent respectively. Comparing Rayleigh method with experiments has varied results for different lengths and materials. The highest difference is 16.834 percent in 100 cm length copper.

Keywords: Natural frequency, Rayleigh method, software, stainless-steel, vibration

Introduction

Vibrations are present in vehicles, motorcycles, musical instruments, airplane wings, and structural waving caused by wind or earthquakes (Inman, 1994). It is a common problem in the field of mechanical engineering. In Universitas Pembangunan Nasional “Veteran” Jakarta, students need to interact with the real phenomenon of simple mechanical vibration to enrich the learning process (Usman, 2022).

Learning mechanical vibration in higher education is usually conducted in laboratory study. Activity at the laboratory or practicum supports students' capability to master fundamental concepts after studying the theory. One indicator of learning outcome is the level of mastering concepts. A cantilever beam or horizontal bending bar is the simplest form of free vibration. The objective of the practicum is to obtain the natural frequency of a horizontal bending bar.

This paper will discuss the design and early test of the apparatus. At the end of the research, we will get a comparison between manual calculation (Rayleigh method), software (Ansys), and experimental. The comparison focused on the natural frequency value of three methods, including three different materials and three length variations.

Literature Study

Finite element method

The finite element method (FEM) solved engineering problems using a numerical approach. The problems are structural analysis, heat transfer, fluid mechanics, and electromagnetic. Other

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than that, FEM can solve geometrical complexity, load type, and material characteristics. As an alternative, this approach creates a mesh consisting of elements and nodes (Lammirta et al., 2018).

Software

Computer-aided-design (CAD) software and finite element methods were used in this research. CAD software was used to draw the beam and complete the overall design. CAD software can validate the work of design and lower the needs of prototypes in future modification (Sasmito, 2018). For the structural analysis, we use ANSYS to model and solve problems in horizontal beams. In this paper, the software solves the natural frequency and maximum displacement (amplitude) of horizontal bars (Isranuri & Firdaus, 2020).

Free vibration of cantilever beam

Vibration is relative movement from a reference position when an object oscillates in one period. Vibration can be expressed as the function of displacement, velocity, or acceleration from that movement (Ali et al., 2019). Another method for approximating the fundamental natural frequency of a vibrating system is using Rayleigh method. An optimization strategy is developed to arrive at the optimum shape function and the results are verified and validated by the finite element method (Wahrhaftig et al., 2022). Although there are some methods to determine the natural frequency, the finite element scheme is an excellent technique for obtaining accurate eigenmodes and mode shapes for similar problems (Kanwal et al., 2024). Effect of physical properties and geometry on the characteristic equation. The research investigates the effects of parameters such as the number of holes, filling ratio, and boundary deformability on the vibration dynamics (Kafkas, 2024).

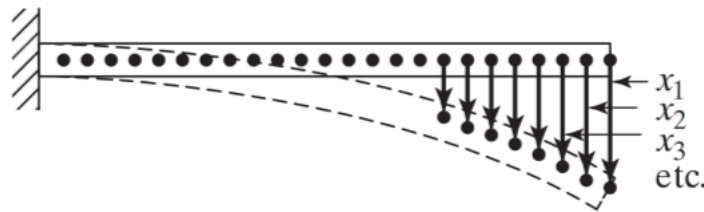


Figure 1. Cantilever beam

The inertial moment of the horizontal bar is the variable to form the equation. The profile of horizontal bar is rectangular, then the equation of inertial moment of bar can be expressed.

$$I = \frac{1}{12}bh^3$$

I is the notation of the inertial moment in m^4 , b is the width of the bar and h is the thickness of the bar.

The stiffness coefficient of horizontal bar as cantilever beam can be expressed.

$$K = \frac{3EI}{L}$$

K is the stiffness coefficient of bar in N/m , E is young's modulus in Pa and L is the length of bar in meters.

To calculate natural frequency (F_n) we can use the free vibration of the cantilever beam. It is a one degree of freedom vibration. The natural frequency can be obtained using.

$$F_n = \sqrt{\frac{k}{\frac{33}{140}M + m}}$$

F_n =natural frequency of bar in Hertz, M is mass of main system and m is the mass of tip of bar.

Experimental of underdamped free vibration

When the damping ratio is more than 0 and less than 1, system is said to be underdamped. The general solution of the governing equation is (Kelly, 2000):

$$x(t) = e^{-\xi\omega_n t} \left[C_1 \cos(\omega_n \sqrt{1 - \xi^2} t) + C_2 \sin(\omega_n \sqrt{1 - \xi^2} t) \right]$$

The free vibrations of an underdamped system are cyclic but not periodic. Even though the amplitude decreases between cycles, the system takes the same amount of time to execute each cycle. This time is called the period of free underdamped vibrations or the damped period and is given by:

$$T_d = \frac{2\pi}{\omega_d}$$

If initial condition, trigonometric identity, energy equation included in the equation resulting logarithmic decrement, δ , is defined for underdamped free vibration as the natural logarithm of the ratio of the amplitudes of vibration on successive cycles.

$$\delta = \ln \left(\frac{x(t)}{x(t + T_d)} \right) = \ln \left(\frac{Ae^{-\xi\omega_n t} \sin(\omega_d t + \phi_d)}{Ae^{-\xi\omega_n(t + T_d)} \sin(\omega_d(t + T_d) + \phi_d)} \right)$$

$$\delta = \xi\omega_n T_d = \frac{2\pi\xi}{\sqrt{1 - \xi^2}}$$

The logarithmic decrement is often measured by experiment and damping ratio determined from:

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

It can be shown that the following equation can also be used to calculate the logarithmic decrement:

$$\delta = \frac{1}{n} \ln \left(\frac{x(t)}{x(t + nT_d)} \right)$$

Damped natural frequency (ω_d) and natural frequency (ω_n) can be determined using:

$$\omega_d = \frac{2\pi}{T_d}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \xi^2}}$$

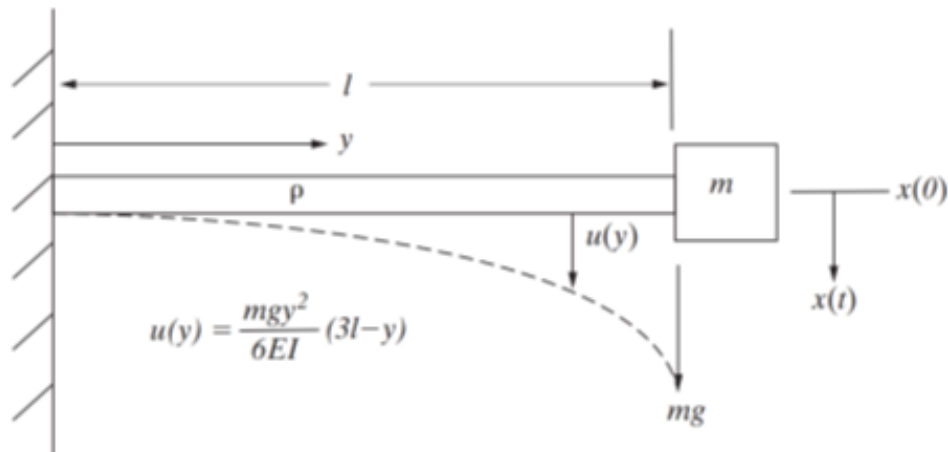


Figure 2. Cantilever Beam and its equation

Material and Methods

Flowcharts

Figure 3 shows the big picture of this research.

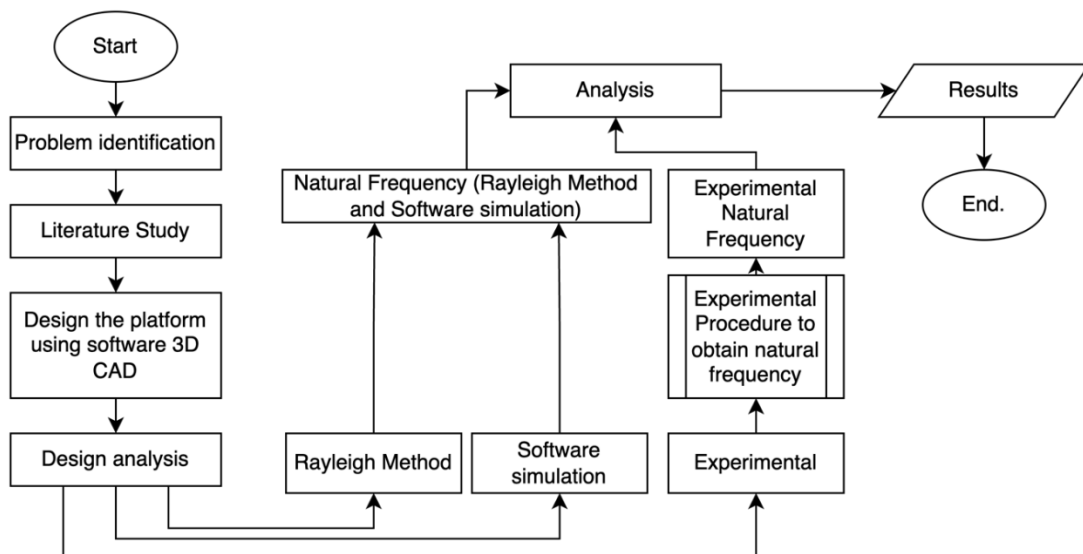


Figure 3. Flowchart

Research variables

Independent variables are values that influence other variables. The first variable is the length, this research uses 100 cm, 90 cm, and 80 cm as length variations. The other variable is material, use young's modulus based on its material: copper, brass, or stainless steel. The three materials data are described in Table 1.

Table 1. Data of bar material

Copper	118	8900
Brass	120	8700

Dependent variables are values that change as a result of independent variables. This research uses natural frequency as a dependent variable.

Experimental

As a subpart of Figure 3, the experiment procedure to obtain natural frequency, Figure 4 shows the sequential works to obtain natural frequency. It starts with finding logarithmic decrements from free vibration and a series of calculations until the natural frequency is acquired.

The platform was built from mild steel hollow 30 x 30 mm in profile, and the thickness is 1.2 mm. A 740x340mm Stainless steel plate was placed on the bottom side of the platform, for better visualization, see Figure 5. The horizontal bar is placed using a clamp in Figure 6 (c). The horizontal bar is 20mm wide and 3mm thick. Each material has three lengths: 100cm, 90 cm, and 80 cm.

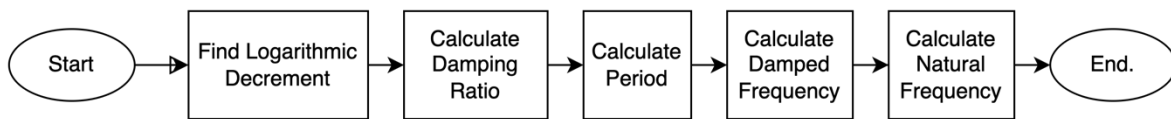


Figure 4. Experiment flowchart

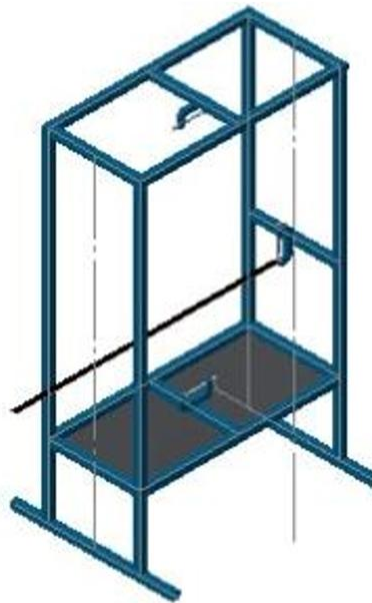


Figure 5. Design of platform using CAD Software

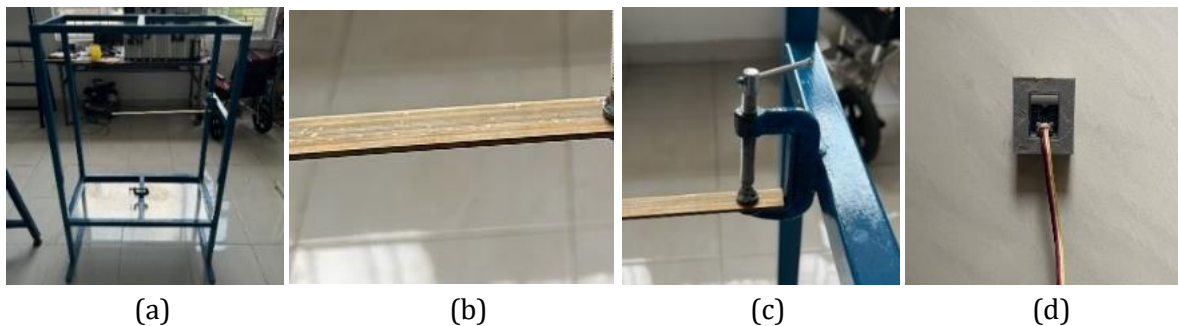


Figure 6. Platform and horizontal bar setup for experimental (a) Platform for testing, (b) bar specimen, (c) clamp at the right end of bar, (d) sensor housing

For data collection, this setup uses MPU9250, an inertial measurement unit 9 axis combined with Arduino uno as the microcontroller. The sensor is placed at the sensor housing at the end of the horizontal bar, it creates a small mass in the system.

Results and Discussion

Design analysis using rayleigh methods

Moment of inertia: using profile 0.02 x 0.003 m of horizontal beam, we can obtain the moment of inertia. The value is $4.5 \times 10^{-11} \text{ m}^4$.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.002)(0.003)^3 = 4.5 \times 10^{-11} \text{ m}^4$$

Stiffness coefficient: Young's modulus (E) varies between three materials of horizontal beam. We use copper, brass and stainless steel with young's modulus consecutively 118 GPa, 120 GPa, and 200 GPa.

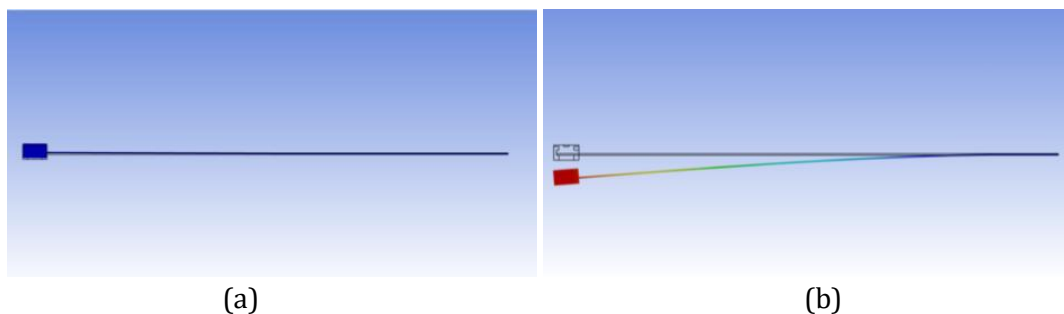
Beam mass and mass at tip of beam: Calculation of mass from density multiply with volume of bar for all material and length shown in Table 2. The highest mass is 100 cm copper bar and lowest mass is 80 cm stainless steel bar.

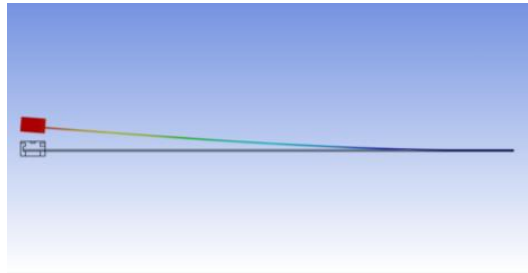
Table 2. The stiffness coefficient of copper, brass and stainless steel varies with its length

Material	Length (cm)	Stiffness Coefficient (N/m)	Mass (Kg)	Natural Frequency (Hz)
Copper	100	15.90705	0.5376	10.2454
	90	21.82037	0.48384	12.5351
	80	31.06846	0.43008	15.6905
Brass	100	16.20	0.5238	10.4521
	90	22.2222	0.47142	12.7855
	80	31.64063	0.41904	16.0001
Stainless Steel	100	27.00	0.462	14.2096
	90	37.03704	0.4158	17.3646
	80	52.73438	0.3696	21.7047

Design analysis using software

The software simulates the movement of the horizontal bending bar. Fixed support placed at the right end of Figure 7(a) to (c). Then a mass placed at the left end of the bar. Table 3 shows the result of natural frequency of all materials and lengths.





(c)

Figure 7. Horizontal bending bar position when at its:
(a) initial position, (b) highest position, (c) lowest position

Table 3. Natural frequency of bending bar, obtained using software and its comparison to Rayleigh method

Material	Length (cm)	Natural Frequency (Hz) (Software)	Natural Frequency (Hz) (Rayleigh)	Difference
Copper	100	9.8992	10.2454	3.379 %
	90	12.1115	12.5351	3,379 %
	80	15.1602	15.6905	3,380 %
Brass	100	9.5391	10.4521	8,735 %
	90	11.6686	12.7855	8,736 %
	80	14.6024	16.0001	8,736 %
Stainless Steel	100	13.9588	14.2096	1,765 %
	90	17.0580	17.3646	1,765 %
	80	21.3215	21.7047	1,765 %

The difference in natural frequency for copper bars between Rayleigh method and software is 3.379 percent. Natural frequency from Rayleigh method higher than software result, shown in Table 3 above. For brass bar, Rayleigh method's natural frequency is 8.735 percent higher than the software simulation lastly, the natural frequency of the stainless-steel bar, Rayleigh method, gets 1.765 percent higher than the software. Overall, the Rayleigh method is always higher than the simulation results.

Experimental results

Table 4. Recapitulation of collecting logarithmic decrement and calculation of damping ratio, period, damped natural frequency and natural frequency

Material	Length (cm)	Logarithmic Decrement δ	Damping ratio ζ	Period (s)	Damped Frequency ω_d (rad/s)	Natural Frequency ω_n (Hz)
Copper	100	0.12136	0.0193	0.55	11.9679	11.9702
	90	0.17069	0.0272	0.45	13.9626	13.9677
	80	0.2336	0.0371	0.35	17.9519	17.9643
Brass	100	0.1224	0.0164	0.6	10.4719	10.4739
	90	0.1848	0.0294	0.5	12.5664	12.5718
	80	0.2409	0.0383	0.4	15.7079	15.7195
Stainless Steel	100	0.1813	0.0288	0.4	15.7079	15.7145
	90	0.3216	0.0511	0.325	19.3328	19.3582
	80	0.4780	0.0758	0.3	20.9439	21.0044

A size of 10 cm used to set initial conditions to create free vibration. MPU9250 sensor with Arduino uno collect data for each material and length. The recapitulation for logarithmic decrement, damping ratio calculation, period, damped natural frequency and natural frequency at Table 4.

Comparison of experimental data, simulation and Rayleigh method. For copper, the highest value of natural frequency is 80cm long bar. And the lowest natural frequency at 100cm. The same trend applies to all materials, so it can be summarized as the shorter a horizontal bar, the higher its natural frequency value.

The difference of natural frequency value between three methods are relatively low, but there are some value gets more than 10 percent difference, that case occurs at copper and stainless-steel material. The highest difference is Rayleigh method versus experiment, it gets 16.834 percent difference. To see the recapitulation, see Table 5. There are some factors that contribute to this more than 10 percent value: quality of material and sensor capability.

To perform better with smaller differences, we can do some tests of material to ensure its quality and match the value of density, young's modulus and other mechanical properties to match software and Rayleigh method. The other factor is the sensor capability, this MPU9250 sensor combined with Arduino uno surely has limitations. The data that we get to find the logarithmic decrement maybe gets some error. For the future, this research needs another method to obtain the tip movement of the horizontal bar, then the difference could be minimized.

Table 5. Comparison of natural frequency value for Rayleigh and experiment method.

Material	Length (cm)	Natural Frequency (Hz)		Difference (%)
		Rayleigh Method	Experiment	
Copper	100	10.2454	9.8992	16.834
	90	12.5352	12.1115	11.428
	80	15.6905	15.1602	14.491
Brass	100	10.4521	9.5391	0.208
	90	12.7855	11.6686	1.671
	80	16.0001	14.6024	1.754
Stainless Steel	100	14.2096	13.9588	10.59
	90	17.3646	17.0580	11.48
	80	21.7047	21.3215	3.226

Conclusion

The length and material of the horizontal bar influence the value of its natural frequency. As the shorter a horizontal bar, the higher its natural frequency value. For the material's natural frequency, it depends on the young's modulus. The higher young's modulus value, the higher the natural frequency of a horizontal bar. The equipment is capable to serve as education laboratory with 16,834 percent difference between Rayleigh method and the experiment.

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